## Time Value of Money Review - Concept Questions

1. What are the four basic parts (variables) of the time-value of money equation? The four variables are present value (PV), time as stated as the number of periods ( n ), interest rate (r), and future value (FV).
2. What does the term compounding mean?

Compounding means that interest is earned on prior interest available in the account.
3. Define a growth rate and a discount rate. What is the difference between them?

A growth rate implies going forward in time, a discount rate implies going backwards in time.
4. What happens to a future value as you increase the interest (growth) rate?

The future value gets larger as you increase the interest rate.
5. What happens to a present value as you increase the discount rate?

The present value gets smaller as you increase the discount rate.
6. What happens to a future value as you increase the time to the future date?

The future value increases as you increase the time to the future.
7. What happens to the present value as the time to the future value increases?

The present value decreases as you increase the time between the future value date and the present value date.
8. What is the Rule of $\mathbf{7 2}$ ?

The Rule of 72 is a method that estimates how long it takes an investment to double in value, or for given a specific time period, what growth rate will double the value of an investment.
9. Is the present value always less than the future value?

Yes, as long as interest rates are positive-and interest rates are always positive-the present value of a sum of money will always be less than its future value.
10. When a lottery price is offered as $\mathbf{\$ 1 0 , 0 0 0 , 0 0 0}$ but will pay out a series of $\mathbf{\$ 2 5 0 , 0 0 0}$ payments over forty years, is it really a $\mathbf{\$ 1 0 , 0 0 0 , 0 0 0}$ lottery prize?
The present value of the lottery is not worth $\$ 10,000,000$. The total payments over time are $\$ 10,000,000$ but this is not a value of the lottery because these payments are at different points in time, and next year's $\$ 250,000$ is not the same as this year's $\$ 250,000$.
11. What is the difference between a series of payments and an annuity? What are the two specific characteristics of a series of payments that make them an annuity?
An annuity is a series of payments of equal size at equal intervals. Uniform payments and equal time intervals such as months, quarters or years, are the two characteristics that make a series of payments an annuity. So, a series of payments can be an annuity but not all series of payments are annuities. If the series of payments is of different values or at different intervals, it is not an annuity.
12. What effect on the future value of an annuity does increasing the interest rate have? Does a change from $4 \%$ to $6 \%$ have the same dollar impact as a change from $6 \%$ to 8\%?
The greater the interest rate the greater the future value of an annuity everything else held constant. Changing the interest rate from $4 \%$ to $6 \%$ will increase the annuity but with a smaller dollar increase when compared to the $6 \%$ to $8 \%$ change.
For example:
Annuity of $\$ 100$ for five years at $4 \%$ : Future Value is $\$ 541.63$
Annuity of $\$ 100$ for five years at $6 \%$ : Future Value is $\$ 563.71$
Annuity of $\$ 100$ for five years at $8 \%$ : Future Value is $\$ 586.66$
Increase from $4 \%$ to $6 \%$ is $\$ 22.08$
Increase from $6 \%$ to $8 \%$ is $\$ 22.95 \ldots$...which is higher
13. What effect on the present value of an annuity does increasing the interest rate have?

Does a decrease from $7 \%$ to $5 \%$ have the same dollar impact as a decrease from $5 \%$ to 3\%?
Decreasing the interest rate (discount rate) increases the present value of an annuity. The impact is different as the discount rates get smaller. For example:
Annuity of $\$ 100$ for five years at 7\%: Present Value is $\$ 410.02$
Annuity of $\$ 100$ for five years at 5\%: Present Value is $\$ 432.95$
Annuity of $\$ 100$ for five years at 3\%: Present Value is $\$ 457.97$
Decrease from 7\% to 5\% increases PV by $\$ 22.93$
Decrease from 5\% to 3\% increases PV by $\$ 25.02$
14. What is the difference between an ordinary annuity and an annuity due?

An ordinary annuity has the payments at the end of the period and an annuity due has the payment due at the start of the period.
15. What does the amortization schedule tell you about a loan repayment?

An amortization schedule tells you the amount of each payment that is applied against the interest expense, the amount applied against the principal and the principal balance after the payment at each scheduled payment.
16. What does it mean that the current principal balance of a loan being repaid as an amortized loan is the present value of the future payment stream?
The outstanding balance or remaining unpaid principal after the application of a scheduled payment reflects the current amount needed to pay off the loan. The remaining scheduled payment stream is another way to pay off the loan. Because both the principal and the remaining scheduled payments are sufficient to pay off the loan the current principal is therefore the present value of the remaining payments.
17. If you increase the number of payments on an amortized loan, does the payment increase or decrease? Why or why not?
Increasing the number of payments (all else held constant) decreases the size of each payment. Reverse logic provides the rationale for the answer. If the payment was increased with the same interest payment, the loan would be paid off sooner with the higher payments. Therefore increasing the time to pay off the loan means the payments have been lowered. As the amount applied to the interest is the same but the amount applied to the principal is lower so it takes more payments to eliminate the principal.
18. If you increase the interest rate on an amortized loan, does the payment increase or decrease? Why or why not?
The payment increases with a rise in interest rates all else held constant. The reason is that more of the payment is applied to the interest and so to reduce the principal at the same pace as before a higher payment is needed.
19. If you won the lottery and had the choice of the lump-sum payoff or the annuity payoff, what factors would you consider besides the implied interest rate (indifference interest rate) in selecting the payoff style?
Factors such as your current wealth or debts could influence your decision. If you have a large amount of debt and want to be debt free you might elect the lump-sum option. If you are terrible at budgeting money and know you would probably squander the money you might elect a slower payment method, i.e. the annuity. If you wanted to be philanthropic you might want to take the lump sum to give more money away to important causes. These are just a few of the potential non- financial impacts that could influence your decision on the pay out style.
20. The concept of time value of money is a recognition that a dollar received today is worth more than a dollar received a year from now, or at any future date. It exists because there are investment opportunities on money; that is, we can place our dollar received today in a savings account and one year from now have more than a dollar.
21. Compounding and discounting are inverse processes of each other. In compounding, money is moved forward in time, while in discounting, money is moved back in time. This can be shown mathematically in the compounding equation:
$\mathrm{FV}_{\mathrm{n}}=\mathrm{PV}(1+\mathrm{r})^{\mathrm{n}}$
We can derive the discounting equation by multiplying each side of this equation by $\frac{1}{(1+\mathrm{r})^{\mathrm{n}}}$ and we get: $\mathrm{PV}=\mathrm{FV}_{\mathrm{n}} \frac{1}{(1+\mathrm{r})^{\mathrm{n}}}$
We know that $\quad \mathrm{FV}_{\mathrm{n}} \quad=\quad \mathrm{PV}(1+\mathrm{r})^{\mathrm{n}}$
Thus, an increase in r will increase $\mathrm{FV}_{\mathrm{n}}$, and a decrease in n will decrease $\mathrm{FV}_{\mathrm{n}}$.
22. Bank C, which compounds continuously, pays the highest interest. This occurs because, while all banks pay the same interest, $5 \%$, bank C compounds the $5 \%$ continuously. Continuous compounding allows interest to be earned more frequently than any other compounding period.
23. An annuity is a series of equal dollar payments for a specified number of years. Examples of annuities include mortgage payments, interest payments on bonds, fixed lease payments, and any fixed contractual payment.
24. A perpetuity is an annuity that continues forever; that is, every year from now on this investment pays the same dollar amount. The difference between an annuity and a perpetuity is that a perpetuity has no termination date, whereas an annuity does. .

