PORTFOLIO OPTIMIZATION WITH SPREADSHEET MODELING: A PEDAGOGICAL APPROACH

Pat Obi, Purdue University Calumet, IN., USA
Raida Abuizam, Purdue University Calumet, IN., USA

ABSTRACT

This paper presents a pedagogical approach for constructing optimal portfolios on spreadsheet. We begin with a review of modern portfolio theory in which the reduction of unsystematic risk is the prime motive for diversification. Three factors impacting overall portfolio risk are asset variances, covariances, and asset weights. Of these three, only the asset weight is at the discretion of the investor. By varying the weights, one observes how the expected performance of a portfolio changes even when the asset composition is unchanged. The spreadsheet analysis begins with a simple illustration of the attainable set and the efficient frontier. Finally, an illustration of portfolio optimization is presented using the solver tool on spreadsheet. The approach described in this paper should be particularly helpful for teaching portfolio management at both the graduate and undergraduate levels. It should also serve as an intuitive method for teaching the concept of risk and return in an MBA-level course in corporate finance.

INTRODUCTION

Security analysis and portfolio management is a two-part concept in investments dealing with portfolio selection. In the first hand, the investor evaluates individual securities for their intrinsic values. Afterward, the investor decides on the fractional amount to invest in each security so as to achieve the desired portfolio performance. An optimal portfolio should provide the investor with the highest expected return given the investor's risk tolerance. Alternatively, the investor should obtain the lowest risk for a given level of expected return. Such a portfolio is considered *efficient* because no other possible combination is more superior either in terms of risk or return. Portfolio managers are able to employ mathematical programming models and proprietary software in order to accomplish this goal for their investors.

In his seminal work entitled *Portfolio Selection*, Markowitz (1952) shows that the variance of a portfolio of N assets can be expressed as:

$$\sigma_P^2 = \sum_{j=1}^{N} w_j^2 \sigma_j^2 + \sum_{j=1}^{N} \sum_{k=1}^{N} w_j w_k \sigma_j \sigma_k \rho_{jk}, \qquad j \neq k \qquad (1)$$

where

$w_{j,k}$ = Proportion invested in each asset (asset weight)

σ_j^2 = Variance of a single asset

ρ_{jk} = Correlation coefficient between pairs of individual assets

Many investments textbooks such as Bodie, Kane, and Marcus (2002) and Elton et al (2003) provide pedagogical insights on the expansion of this model. As an example, the variance of a three-asset portfolio is the weighted sum of the variance and covariance of the three assets in the portfolio, as follows:

$$\sigma_P^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_3^2 \sigma_3^2 + 2 w_1 w_2 \sigma_{12} + 2 w_1 w_3 \sigma_{13} + 2 w_2 w_3 \sigma_{23} \quad (2)$$

where

σ_{jk} = $\sigma_j \sigma_k \rho_{jk}$ = Asset covariance

The expected return on a three-asset portfolio is the weighted average of the mean returns ($\bar{r}_j$) of the three assets::

$$\mu_P = w_1 \bar{r}_1 + w_2 \bar{r}_2 + w_3 \bar{r}_3$$

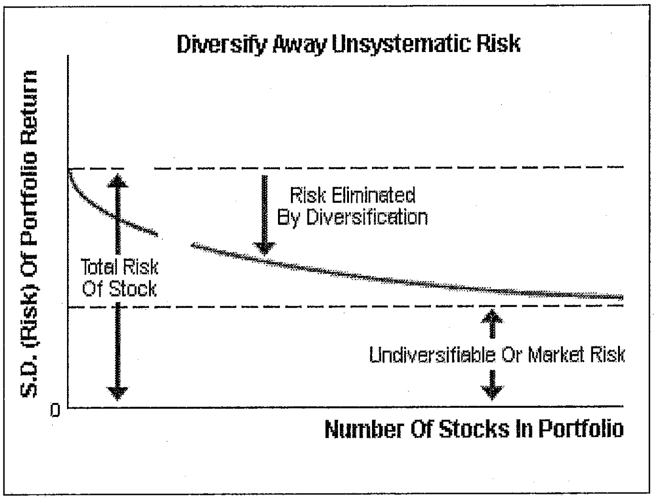
As Equation 2 shows, there are three distinct factors that influence portfolio risk: variances of the individual securities (σ^2_j), covariance (or correlation coefficient) between the securities (σ_{jk}), and each asset's investment fraction (w_j). While variance and covariance are determined by factors beyond the investor's control, the asset weight is a choice that is at the discretion of the investor. Therefore, given the asset variances and covariances, the investor can directly control portfolio risk by his or her choice of asset weights.

In choosing a portfolio over a single asset, the prime objective function is risk reduction, which is accomplished by investing in a well diversified portfolio. It is important to note that the only risk that can be reduced is *unsystematic risk*, because it is specific to the firm. Systematic risk (also called market risk) is non-diversifiable. Note that an investor need not invest in a portfolio in order to maximize expected return. A single security might indeed have an expected return higher than that of a portfolio. Nevertheless, an optimal portfolio is one that provides the highest expected return for a given level of risk, or equivalently, the lowest risk for the same level of return.

Rational investors are able to minimize unsystematic risk by investing in a diversified portfolio. A diversified portfolio is one in which the asset covariances are low. This means that the underlying firms have very little in common in terms of their specific operations. Many finance textbooks are able to expatiate on this concept by presenting examples of how differences in asset covariance directly impact portfolio risk. What is not often clear in these illustrations is the risk impact of asset weight and its linkage with the *efficient frontier*.

Bagamery and Johnson (2006) consider the difficulty that students have in comprehending the construct of the portfolio variance. They point out that although many textbooks emphasize asset covariances as key to diversification, individual security variances also play a critical role in determining the overall portfolio variance. This is because in addition to the variance terms that appear in Equation 1, asset variances are also incorporated in the covariance terms shown in Equation 2. However McClure (2006) contends that this argument is of little consequence when considering the variance of a well diversified portfolio. In his illustration, presented as Figure 1, McClure explains that it is rather the difference between the levels of risk of individual stocks (i.e. their covariance) that primarily influences the overall portfolio risk. It is in this sense that investors benefit from holding diversified portfolios instead of individual stocks.

Figure 1: Portfolio Diversification



Using a spreadsheet approach, this paper presents a pedagogical path for understanding the significance of asset weights in the construction of optimal portfolios. The spreadsheet illustration shows that while it is possible for different investors to posses the same portfolio of assets, the performance of the portfolios may still differ even with the same variances and covariances. By this approach, students are able to see that what makes for an optimal portfolio is not so much of the variety of assets within the portfolio but increasingly, by the choice of each asset's weight. In the extended illustration, we show how the spreadsheet solver tool is used to calculate optimal portfolio weights. The optimal weights are then used to calculate the minimum variance portfolio as well as the corresponding mean return. In this way, students are able to not only actively learn the theory behind the efficient frontier but are also able to apply spreadsheet modeling in making rational portfolio choices.

BACKGROUND

Modern Portfolio Theory (MPT) provides a framework whereby risk-averse investors are able to diversify their investments in order to optimize their portfolio performance. Optimal portfolios are those that provide the highest expected return for a given level of risk or, the lowest risk for a given expected return. Further, MPT sets out a path for the pricing of a security given its risk relative to the market as a whole. The basic concepts of the theory are described in the seminal work of Markowitz (1952) and extended by Sharpe's (1964) Capital Asset Pricing Model (CAPM). For their pioneering work in the development of MPT, these two scholars were co-recipients of the 1990 Nobel Prize in Financial Economics.

In choosing optimal portfolios, Markowitz shows that risk-averse investors expect to be compensated for taking additional risk on the expected returns-variance of returns (E-V) space. The algorithm used to generate the E-V frontier is known as mean-variance optimization, since what is being optimized is expected return versus variance. Anderson and Jones (2005) explain that which portfolio an investor ultimately chooses on the E-V space is a function of three interrelated factors: investment horizon, relative risk aversion, and the size of investment.

In Modern Portfolio Theory, the total risk of a security is measured by variance or standard deviation. An investor can reduce the unsystematic risk component of a portfolio simply by holding assets that are less than perfectly correlated. Systematic risk, measured by beta, is non-diversifiable and is the only risk factor that is priced into a security. Portfolio beta is the weighted average of the betas of the individual assets within the portfolio.

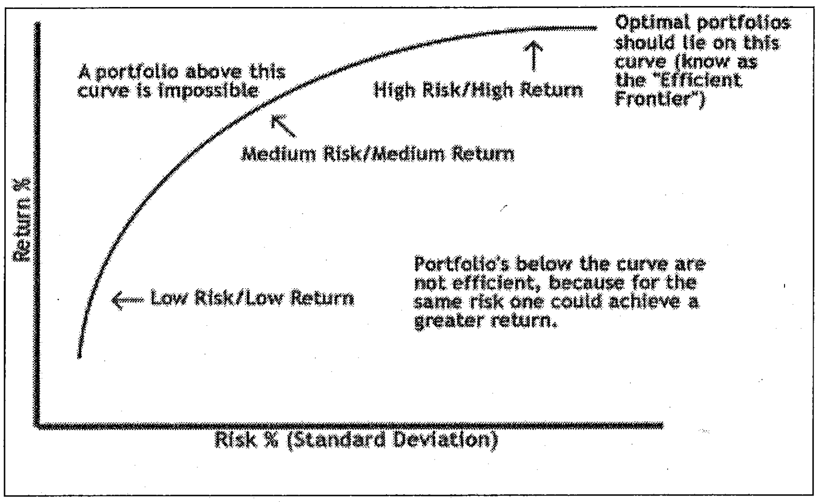
The mass appeal of Modern Portfolio Theory has not gone unchallenged. Swisher and Kasten (2005) argue that the reliance of MPT on variance as a measure of total risk may be misleading for two reasons: First, security returns are not normal. This means that upside deviations from the mean differ from downside deviations. Second, variance fails to describe human risk, an emotional condition that rational investors factor into their buy-and-sell decisions. Swisher and Kasten then offer the *post-modern portfolio theory* (PMPT) as a useful alternative. In it, they propose an investor-specific minimal acceptable return, which accounts for not only MPT's downside variance risk but also contains attributes of investor behavior. It is noteworthy that empirical tests of the two models show insignificant differences in risk measurement. Frank Sortino and Stephen Satchell present a collection of writings on the subject of risk and the post-modern portfolio theory entitled *Managing Downside Risk in Financial Markets*.

All in all, Fabozzi, Gupta and Markowitz (2002) explain that MPT is simply a normative theory in that it describes a standard or norm of behavior that investors should pursue in choosing their portfolios. In contrast, the CAPM is a positive theory because it hypothesizes how investors behave rather than how they should behave.

CONSTRUCTING THE EFFICIENT FRONTIER

The efficient frontier is a set of optimal portfolios plotted on the E-V space as shown in Figure 2. Optimal portfolios have the highest expected return for a given level of risk as measured by standard deviation. Alternatively, they have the lowest risk for a given level of return. The minimum variance portfolio lies on the southwestern tip of the frontier. According to the MPT, rational investors select portfolios that lie on the frontier pursuant to each investor's risk tolerance. The key assumption in MPT is that investors are risk averse in that given two assets that offer the same return, they would prefer the less risky one. Thus, an investor will take on increased risk only if compensated by higher expected returns. Conversely, an investor who wants higher returns must accept more risk. The exact trade-off differs by the investor's degree of risk aversion.

Figure 2: Efficient Frontier



A SPREADSHEET ILLUSTRATION

How does one construct the efficient frontier? Although quite a few published works provide a variety of spreadsheet illustrations of the efficient frontier, the approach in this paper differs in one major respect. It suggests a simple pedagogical model that brings to light why the same set of assets held in differently weighted portfolios possess different risk-return outcomes. The pedagogical approach is outlined as follows:

Step 1. Identify a set of securities for inclusion in the portfolio.

We begin with a three-asset portfolio comprising the following securities:

- Coors Brewing Company
- General Electric
- Equity index (S&P 500)

Step 2. Obtain monthly price data, adjusted for dividends and splits.

We use historical data for 7 years, from January 1995 to December 2001.

- Calculate the monthly rates of return for each asset

- Using the monthly returns, calculate summary statistics for risk and return. Probability data may also be used to estimate risk and return parameters. Historical data are typically sufficient if the investor believes that the future performance of the firms would not deviate significantly from past performance. Results are presented in Table 1.

Table 1: Risk and Return Parameter Estimates on Spreadsheet +

	Index	GE	Coors
Sample mean return: $\bar{r}$ Spreadsheet function: =AVERAGE(cells)	1.19%	2.14%	1.91%
Sample variance: s^2 Spreadsheet function: =VAR(cells)	0.0021	0.0049	0.0070
Sample standard deviation: s Spreadsheet function: =STDEV(cells)	4.59%	7.03%	8.39%

+ Results are based on monthly data

Step 3. Determine the diversification potential of the portfolio by calculating the correlation
matrix of returns.

- In the TOOLS menu on Excel spreadsheet go to DATA ANALYSIS
- Use the CORRELATION dialog box to obtain the correlation matrix of returns. Results are presented in Table 2.

Table 2: Correlation Matrix of Returns

	Index	GE	Coors
Index	1.0000	0.7614	0.1004
GE	0.7614	1.0000	0.0930
Coors	0.1004	0.0930	1.0000

Students may notice that the low correlation coefficients between Coors and GE (0.09) and between Coors and the market index (0.10) suggest that this portfolio is reasonably diversified.

Step 4. Calculate each stock's beta using regression.

The monthly returns for each stock (Coors and GE) are regressed against the monthly returns on the equity index (S&P 500). Stock returns are dependent (Y) variables. The slope of the regression line is the stock's beta estimate.

- In the TOOLS menu go to DATA ANALYSIS
- Use the REGRESSION dialog box to obtain the regression results, as follows:

GE Beta	1.17
Coors Beta	0.18

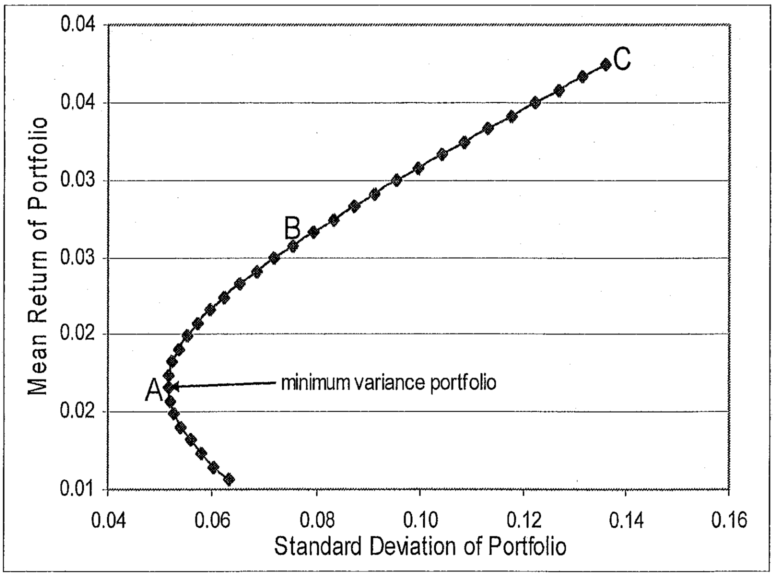
Since market beta is 1, GE's beta of 1.17 suggests that its market risk contribution within the portfolio is slightly higher than that of the stock index. On the other hand, Coors' low beta of 0.18 means that it is hardly influenced by market shocks. As a result, Coors' market risk contribution within the portfolio is much less than that of either GE or the market index.

Step 5. Construct the efficient frontier.

Begin by calculating the mean and variance of the 3-asset portfolio as follows:

- For each set of weights chosen, calculate portfolio mean and portfolio variance as described in Equation 2. Then calculate the portfolio standard deviation by taking the square root of the variance using the SQRT function
- Allowing for short sale (negative weights) is optional
- To obtain a smoothed efficient frontier with a bullet shape, increment the weights by a constant amount
- Construct the efficient frontier by plotting an XY (Scatter) graph of the standard deviation (X-axis) and the mean (Y-axis). To do this, use the Chart Wizard as follows: highlight the two data columns containing standard deviation and mean; next click on Chart Wizard; finally select the XY graphing option.
- Figure 3 shows the efficient frontier. The dataset used to construct the graph is presented in Appendix 1.

Figure 3. Efficient Frontier of a Three-Asset Portfolio



Each point on the graph represents a portfolio containing the same set of assets (GE, Coors, and the stock index). However the risk and return characteristics of each portfolio differ by the weight combination. Therefore, unless mutual funds professing to invest in the same stocks (such as equity index funds) maintain identical asset weights, the performance of the funds would necessarily vary over the same horizon.

In Figure 3, portfolio combinations A, B, and C are all efficient. The *minimum variance* portfolio A has the lowest risk compared to any other portfolio with the same return. Portfolio C is the maximum return portfolio, based on the weights selected. Portfolios plotting below A, B and C, inclusive, are inferior in that there is some asset combination on the frontier which, for the same risk, promises a higher return.

MODELING WITH SPREADSHEET SOLVER

The optimization model on spreadsheet uses the solver tool to identify optimal portfolios. The model inputs, summarized in Tables 1 and 2, are the individual asset means, standard deviations, and the correlation or covariance matrix. The solver dialog box is then used to calculate the investment fractions or weights within the decision cells. The optimal weights are those that calculate the minimum variance (or maximum return) portfolio.

Model Constraints

The primary constraint specified in the Solver table is that the sum of the asset weights be equal to 1. Additionally, one might specify that the portfolio mean return, calculated with the optimal weights, be at least equal to the *required return* on the portfolio. One way to estimate the required rate of return on the portfolio (r_P) is to use the Capital Asset Pricing Model (CAPM). Using the CAPM, the required rate of return is calculated as the Security Market Line (SML): $r_P = r_F + \beta_P(r_M - r_F)$, where r_F is the riskfree interest rate; r_M is the market return; and β_P is the portfolio beta. The model is specified as a nonlinear model in its objective function although the constraints are linear.

The Procedure and Output

Table 3 presents Solver input/output cells for the case where short sale is excluded. First, initial weights are stipulated for each asset on the spreadsheet. The sum of these weights is calculated in the *sum* cell to be equal to 1. With these weights, the portfolio variance (or standard deviation) and the portfolio mean are calculated – manually – in their respective cells. It is optional to specify the required rate of return on the portfolio.

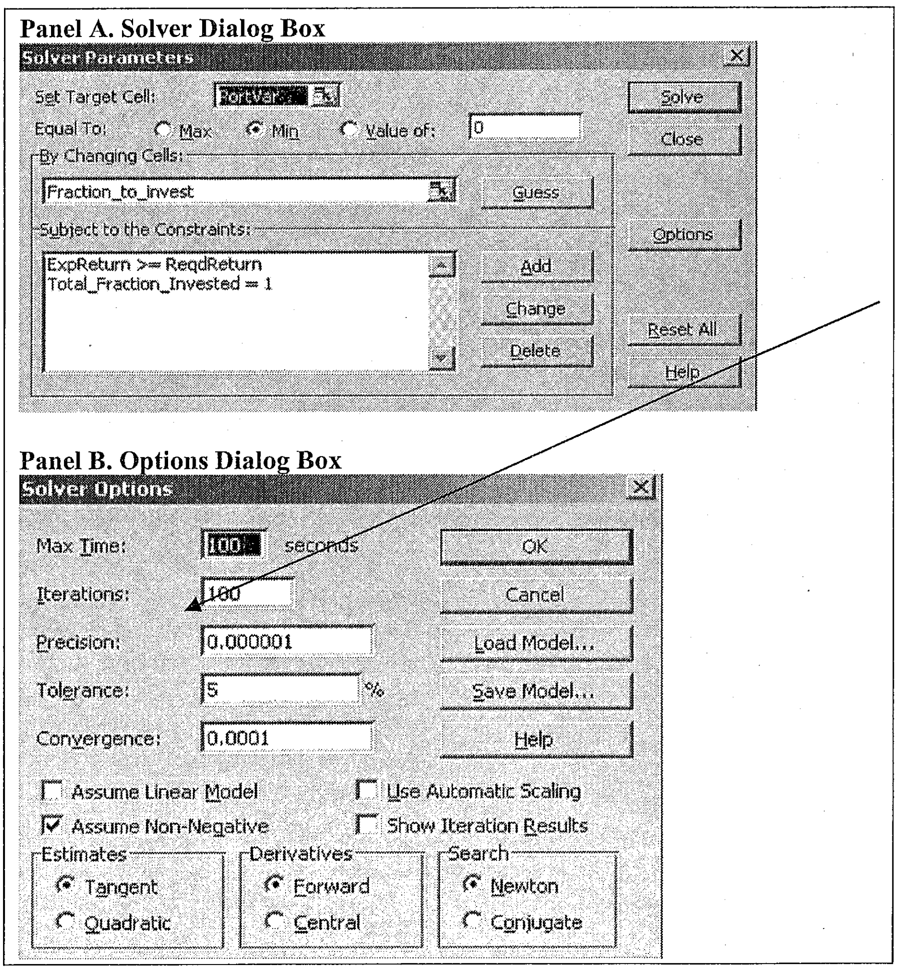
Table 3: Model Input/Output in the Absence of Short Sale

	Index	**GE**	**Coors**	**Sum**
Optimal weights +	0.7946	0.0000	0.2054	1
		Comment		
Variance of portfolio	0.001753	Optimized by Solver		
Standard deviation of portfolio	0.041874	Calculated		
Mean portfolio return	0.013379	Solved		
Required return of portfolio	0.010102	Specified		

+ specify any set of weights initially in the respective cells for each asset before the run

The Solver dialog box is located in the Tools menu on spreadsheet. The optimization process is illustrated in Figure 4 for the case where short sale (i.e. negative weight) is excluded. Appendix 2 illustrates the case where short sale is allowed. A short sale allows the investor to borrow against any of the assets. In which case, it is possible for any of the asset weights to be negative. The Target Cell references the cell containing the portfolio variance on the spreadsheet. The Min option is checked for the minimum variance portfolio. The choice variables in any portfolio optimization are the individual asset weights. Therefore, the cells containing the asset weights on spreadsheet should be identified as the Changing Cells. The constraints are stipulated by clicking the Add button and then completing the query. The Options button in Panel A opens the dialog box shown in Panel B. This allows you to check the Assume Non-Negative option, if you do not wish to allow for short sale. The Options button in Panel A should be ignored if short sale is to be allowed. The data contained in Table 3 are the optimized results produced by Solver.

Figure 4: Portfolio Optimization on Spreadsheet



As shown in Table 3, the model suggests that the minimum variance portfolio calls for investing about 79.5 percent of investor funds in the market index and the remainder in Coors. No funds should be allocated to GE. With this asset mix, the standard deviation of the portfolio is 4.2 percent and the expected portfolio return is 1.33 percent, per month. It is noteworthy that this mean return is greater than the specified required monthly return of 1 percent, which satisfies the secondary model constraint. Is this solver solution optimal? Winston and Albright (2001) explain that when the model constraints are linear as stipulated, it can be shown that the portfolio variance is a convex function of the asset weights. As such, one can conclude that the solver solution is optimal.

DISCUSSION

Portfolio optimization is the core concept in portfolio theory. In the investment services industry, portfolio managers use a variety of proprietary software to determine optimal weights when rebalancing their portfolios. The aim is to ensure that the stipulated risk-return goal for the mutual fund is pursued. From a pedagogical standpoint, spreadsheet modeling offers students an opportunity to more easily capture the essence of portfolio optimization. But more importantly, it offers them the path to gain a hands-on experience in the calculation of portfolio variance as well as the determination of optimal portfolios.

Many investments textbooks come with author-designed analytical software that allows students to calculate efficient portfolios. Many of these software are spreadsheet based. The downside to these is that they are canned analytical packages that deny students the opportunity to learn the underlying algorithm. Although many aspects of spreadsheet modeling are well known to business students, in particular Finance majors, the logical process presented in this paper should facilitate the instructor's ability to actively demonstrate the construction of optimal portfolios. And for the student, it provides an intuitive path for identifying the set of optimal portfolios that satisfy the investor's risk-return objective function.

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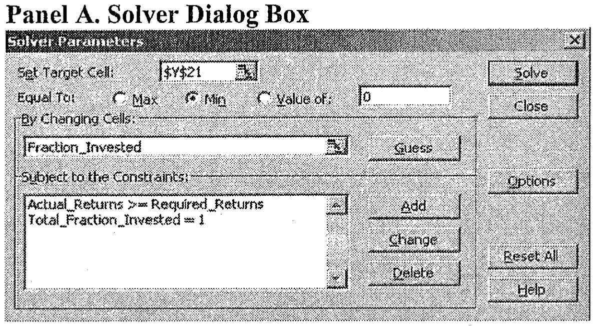
Appendix 1: Data for the Construction of the Efficient Frontier

σ^2_p	σ_p	μ_p	W_{INDEX}	W_{GE}	W_{COORS}	Weight Total
0.0185	0.1361	0.0375	-2.00	1.70	1.30	1.00
0.0173	0.1314	0.0366	-1.90	1.65	1.25	1.00
0.0161	0.1267	0.0358	-1.80	1.60	1.20	1.00
0.0149	0.1221	0.0350	-1.70	1.55	1.15	1.00
0.0138	0.1176	0.0341	-1.60	1.50	1.10	1.00
0.0128	0.1130	0.0333	-1.50	1.45	1.05	1.00
0.0118	0.1086	0.0324	-1.40	1.40	1.00	1.00
0.0108	0.1041	0.0316	-1.30	1.35	0.95	1.00
0.0100	0.0998	0.0308	-1.20	1.30	0.90	1.00
0.0091	0.0955	0.0299	-1.10	1.25	0.85	1.00
0.0083	0.0913	0.0291	-1.00	1.20	0.80	1.00
0.0076	0.0872	0.0283	-0.90	1.15	0.75	1.00
0.0069	0.0831	0.0274	-0.80	1.10	0.70	1.00
0.0063	0.0792	0.0266	-0.70	1.05	0.65	1.00
0.0057	0.0755	0.0257	-0.60	1.00	0.60	1.00
0.0052	0.0719	0.0249	-0.50	0.95	0.55	1.00
0.0047	0.0685	0.0241	-0.40	0.90	0.50	1.00
0.0043	0.0653	0.0232	-0.30	0.85	0.45	1.00
0.0039	0.0623	0.0224	-0.20	0.80	0.40	1.00
0.0036	0.0596	0.0215	-0.10	0.75	0.35	1.00
0.0033	0.0573	0.0207	0.00	0.70	0.30	1.00
0.0031	0.0553	0.0199	0.10	0.65	0.25	1.00
0.0029	0.0537	0.0190	0.20	0.60	0.20	1.00
0.0028	0.0526	0.0182	0.30	0.55	0.15	1.00
0.0027	0.0520	0.0174	0.40	0.50	0.10	1.00
0.0027	0.0518	0.0165	0.50	0.45	0.05	1.00
0.0027	0.0521	0.0157	0.60	0.40	0.00	1.00
0.0028	0.0529	0.0148	0.70	0.35	-0.05	1.00
0.0029	0.0542	0.0140	0.80	0.30	-0.10	1.00
0.0031	0.0559	0.0132	0.90	0.25	-0.15	1.00
0.0034	0.0580	0.0123	1.00	0.20	-0.20	1.00
0.0037	0.0605	0.0115	1.10	0.15	-0.25	1.00
0.0040	0.0632	0.0106	1.20	0.10	-0.30	1.00

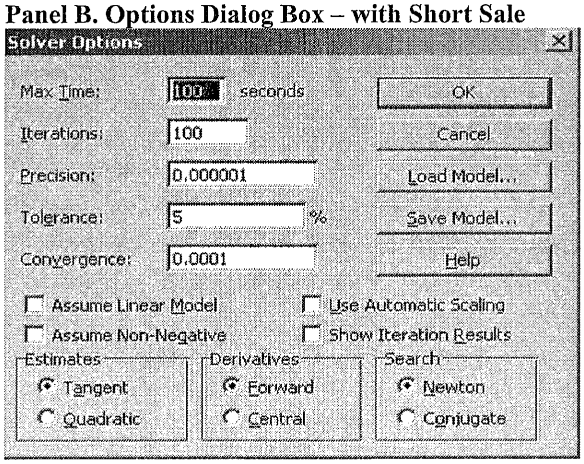
	Index	GE	Coors
Sample mean return: $\bar{r}$	1.19%	2.14%	1.91%
Sample standard deviation: s	4.59%	7.03%	8.39%

Appendix 2: Portfolio Optimization on Spreadsheet

Panel A. Solver Dialog Box



Panel B. Options Dialog Box – with Short Sale



To allow short sale, there is no check on *Assume Non-Negative*

About the Authors:

Pat Obi, Purdue is Professor of Finance in the Department of Finance and Economics at Purdue University Calumet, Hammond, Indiana, USA. He can be reached at obi@calumet.purdue.edu

Raida Abuizam is Assistant Professor of Operations Management in the Department of Finance and Economics at Purdue University Calumet, Hammond, IN., USA. She can be reached at abuizam@calumet.purdue.edu